Cosmological Solutions of Type II String Theory[*]

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Abstract

We study cosmological solutions of type II string theory with a metric of the Kaluza–Klein type and nontrivial Ramond–Ramond forms. It is shown that models with only one form excited can be integrated in general. Moreover, some interesting cases with two nontrivial forms can be solved completely since they correspond to Toda models. We find two types of solutions corresponding to a negative time superinflating phase and a positive time subluminal expanding phase. The two branches are separated by a curvature singularity. Within each branch the effect of the forms is to interpolate between different solutions of pure Kaluza–Klein theory.

Presently, one of the main challenges in string theory is to find solutions compatible with the standard picture of early-universe cosmology. Though successful in reproducing most generic features of low-energy particle phenomenology string theory poses serious problems in realizing even basic ingredients of cosmology such as inflation.

So far, most studies of string cosmology were concerned with solutions of the heterotic string effective action [1]. The basic picture is provided by the classical "rolling–radii"–solutions of Mueller [2, 3] which are Kaluza–Klein–type [4] solutions with time-dependent dilaton and radii of the internal and external space. Classical solutions with a nontrivial Neveu–Schwartz (NS) 2–form have also been found [5] and the $O(d, d)$ symmetry of the low energy effective action proves to be useful in their classification [6].

With the discovery of string dualities [7] the general picture of string theory has changed dramatically. The five formerly unrelated consistent string theories are now believed to correspond to certain limiting cases of one underlying theory, called M–theory whose effective low-energy Lagrangian is given by 11–dimensional supergravity [8]. Correspondingly, 11-dimensional and type II supergravities may be of direct importance for particle phenomenology as well as cosmology [9] and a study of cosmological solutions in these theories appears to be an important issue.

A large amount of work has been devoted to the study of classical solutions in type II theories, however almost all of this work is concentrated on black hole and membrane solutions [10]. The characteristic property of these solutions which have played a major rôle in uncovering string dualities is that they are charged with respect to the Ramond–Ramond (RR) forms of type II string theory.

In this letter we present a first study of cosmological solutions in type II theories with nontrivial RR–forms. The main ideas will be illustrated by a few examples. A systematic study of this class of cosmological solutions will be presented elsewhere [11].

We consider a D–dimensional space time (with $D = 10$ in most examples) of Kaluza–Klein type split up into a number of maximally symmetric flat subspaces of dimensions d_i with scale factors $a_i = \exp(\alpha_i)$. This choice along with a time dependent dilaton ϕ appears to be the simplest appropriate to cosmology and it is remarkably close to the space–times corresponding to membrane solutions. The rôle of the coordinates transverse to the membrane

is here played by time and in contrast to real membranes the "worldsheet" is purely spacelike. Also the two types of membrane solutions – elementary and solitonic – find their natural analog in our cosmological setting : the symmetry of the Kaluza–Klein space is compatible with two types of field strengths one having a nontrivial time–direction (elementary) the other one being nonvanishing in spatial directions only (solitonic).

As we will see the forms provide an effective potential for the scale factors α_i and the dilaton which depends on the type of forms that are considered and the specific Ansatz. It generates interesting dynamics of the model beyond what is known for the pure Kaluza–Klein case [2]. Fortunately, general solutions can be found as long as only one form is turned on. This will be demonstrated below for a type IIB example with a nontrivial RR 2–form. For more than one form this is no longer generally possible. However, as we will show, a number of interesting cases correspond to Toda–models [12, 13] which can be integrated completely. The use of Toda–theory in finding Kaluza–Klein dyon solutions has been advocated in ref. [14] and it has first been applied to cosmological models in ref. [15]. Applications to cosmologies with perfect fluids can be found in ref. [16]. Recently, also nonextremal soliton solutions in supergravity have been studied using Toda theory [17].

We will present examples within IIA supergravity having an elementary RR 3–form and a solitonic RR 1–form or an elementary NS 2–form) leading to a $SU(2)^2$ or $SU(3)$ Toda theory respectively.

The following general picture emerges form the study of these solutions. In all cases we find two types of solutions corresponding to a negative $(-)$ and positive $(+)$ time branch. Except for specific initial conditions these two branches are separated from each other by a curvature singularity. For both branches we can find certain limiting cases in comoving time t (such as $t \to \pm\infty$, t close to the singularity or in some intermediate range) where the forms are effectively turned off and the theory behaves like a pure Kaluza–Klein theory with a time-dependent dilaton, i. e. it is described by Hubble parameters

$$
\begin{aligned}
H_i &= \dot{\alpha}_i \simeq \frac{p_i}{t} \\
H_\phi &= \dot{\phi} \simeq \frac{p_\phi}{t}
\end{aligned}
\tag{1}
$$

satisfying the constraints

$$\sum_{i=1}^{n} d_i p_i = 1$$

$$\frac{1}{2}p_\phi^2 + \sum_{i=1}^{n} d_i p_i^2 = 1 . \tag{2}$$

In particular, these constraints imply that $|p_i| < 1$ always. For the $(+)$ branch $(t > 0)$ this results in a subluminal, "radiation like" expansion (p_i positive) or contraction (p_i negative). For the $(-)$ branch $(t < 0)$ the space will be superinflating for $p_i < 0$ (the horizon shrinks) or collapsing for $p_i > 0$. Though the properties of the $(-)$ branch are very similar to the superinflating phase of pre–big–bang models [18] they arise in the Einstein frame as opposed to the string frame which is used in those models.

The forms become operative during the transition periods between the above asymptotic regions. Their effect can be described by a mapping $p_i \rightarrow \tilde{p}_i$ changing the expansion coefficients p_i in one asymptotic region to $\tilde{p}_i$ in another asymptotic region. For example a universe in the $(+)$ branch, split such that $(d_1, d_2) = (3, 6)$, with a contracting 3–dimensional and an expanding 6–dimensional subspace at early times t, can be turned into the "desired" one with expanding 3–dimensional and contracting 6–dimensional subspace at late times t.

Let us now be more specific and discuss some explicit examples. The action in the Einstein frame is of the general form

$$S = \int d^D x \, \sqrt{-g} \left[R - \frac{4}{D-2}(\partial\phi)^2 - \sum_r \frac{1}{2(\delta_r + 1)!} e^{-p_r\phi} F_r^2 \right] \tag{3}$$

with the D–dimensional metric g_{MN}, the dilaton ϕ and a number of form fields $F_r = dA_r$, $r = 1...m$ of degree δ_r. For type II theories the dilaton couplings p_r are given by

$$p_r = \begin{cases} \frac{8}{D-2} & \text{NS} \quad 2-\text{form} \\ \frac{4\delta_r - 2(D-2)}{D-2} & \text{RR} \quad \delta_r-\text{form} \end{cases} . \tag{4}$$

Examples in $D = 11$ supergravity can be accommodated by setting $\phi = $ const. In eq. (3) we have neglected Chern–Simons terms for simplicity and

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our examples will be chosen such that this is consistent. Other solutions with nonvanishing Chern–Simons terms can presumably be generated from ours via duality [19].

For our first example we will use the simplest split of space $(d_1, d_2) = (3, 6)$ and a solitonic type IIB RR 2–form F in the $d_1 = 3$ subspace. Correspondingly, the Ansatz for the metric, the 2–form and the dilaton reads

$$
\begin{aligned}
ds^2 &= -N^2(\tau)d\tau^2 + e^{2\alpha_1} \sum_{\mu=1}^{3} dx_\mu^2 + e^{2\alpha_2} \sum_{m=4}^{9} dx_m^2 \\
F_{\mu\nu\rho} &= e^{-6\alpha_1} u\, \epsilon_{\mu\nu\rho} \\
\phi &= \phi(\tau) \, ,
\end{aligned}
\tag{5}
$$

where u is a constant [1]. Using the gauge $N = \exp(3\alpha_1 - 6\alpha_2 - \phi)$ we arrive at the following equations of motion

$$
\begin{aligned}
\frac{d}{d\tau}\left(e^{12\alpha_2+\phi}(\alpha_1' + 3\alpha_2')\right) &= 0 \\
\frac{d}{d\tau}\left(e^{12\alpha_2+\phi}(3\alpha_1' + 5\alpha_2')\right) - \frac{u^2}{2} &= 0 \\
\frac{d}{d\tau}\left(e^{12\alpha_2+\phi}\phi'\right) + \frac{u^2}{2} &= 0 \\
e^{12\alpha_2+\phi}(6\alpha_1'^2 + 36\alpha_1'\alpha_2' + 30\alpha_2'^2 - \tfrac{1}{2}\phi'^2) - \frac{u^2}{2} &= 0
\end{aligned}
\tag{6}
$$

for α_1, α_2, $\phi = \alpha_\phi$ and N, respectively. The prime denotes the derivative with respect to τ. Their general solution is given by

$$
\alpha_I = c_I \ln(\tau_1 - \tau) + w_I \ln\left(\frac{\tau}{\tau_1 - \tau}\right) + k_I
\tag{7}
$$

where the index $I = (i, \phi)$ labels the scale factors and the dilaton. The integration constants w_I and k_I are subject to the constraints

$$
\begin{aligned}
12w_2 + w_3 &= 1 \\
12w_1^2 + 72w_1 w_2 + 60w_2^2 &= w_3^2 \\
\exp(12k_2 + k_3) &= u^2 \, .
\end{aligned}
\tag{8}
$$

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[1] For the totally antisymmetric tensor we use the convention $\epsilon^{123} = 1$ and $\epsilon_{\mu\nu\rho} = g_{\mu\alpha}g_{\nu\beta}g_{\rho\gamma}\epsilon^{\alpha\beta\gamma}$.

The range of the time parameter τ is specified by $0 < \tau < \tau_1$. It is remarkable that the numerical coefficients

$$(c_1, c_2, c_\phi) = \frac{1}{8}(-3, 1, 4) \tag{9}$$

in front of the first term of the solution (7) coincide with those of a solitonic 5–brane solution [10]. In fact, this first term represents the analog of such a solution, which is characterized by the proportionality of α_1, α_2 and ϕ. This raises the immediate question of whether cosmological BPS solutions which preserve one half of the supersymmetries can be found. In the case at hand, as well as in the other 10–dimensional examples with one form excited, this turns out to be impossible since the second term in eq. (7) cannot be consistently set to zero. In fact, inspection of the supersymmetry transformation shows that this is precisely what is required for a supersymmetric solution. Still, the form of eq. (7) is so tantalizingly close to allowing a BPS solution that one might hope for such a possibility in related models.

To discuss the cosmological properties of this model it is useful to have expressions for the gauge parameter N and the curvature :

$$N \sim (\tau_1 - \tau)^{-x-\Delta-1}\,\tau^{x-1} \tag{10}$$
$$R \sim (\tau_1 - \tau)^{2(x+\Delta)}\,\tau^{-2x}P(\tau) \ . \tag{11}$$

The quantities x, Δ are given by

$$\begin{aligned} x &= 3w_1 + 6w_2 \\ \Delta &= \frac{3}{8} \end{aligned} \tag{12}$$

and P denotes a second-order polynomial in τ. We remark that the comoving time t can be explicitly expressed in terms of τ by integrating $dt = N(\tau)d\tau$ leading to hypergeometric functions. The constraints (8) admit two ranges for the parameter x, $x > 0$ and $x < -\Delta$. This implies the following mapping of ranges

$$\tau \in [0, \tau_1] \ \rightarrow \ t \in \begin{cases}]-\infty, t_1] & \text{for} \ \ x < -\Delta \ , \quad -\text{branch} \\ [t_0, +\infty[& \text{for} \ \ x > 0 \ , \qquad +\text{branch} \end{cases} \tag{13}$$

between τ and the comoving time t. Since the intermediate region $-\Delta < x < 0$ is forbidden there is no way to connect the two branches within this class

of models. The above expression for R shows that generically there will be a curvature singularities at the finite ends of both branches. For special initial conditions, however, the polynomial P can partially cancel the prefactors in eq. (11) and the curvature singularity disappears. This occurs for the $(-)$ branch if $w_3 = c_3$ and $x \geq -\Delta - 1/2$ and for the $(+)$ branch if $w_3 = 0$ and $x \leq 1/2$. Such a cancellation is very similar to what happens in the singularity-free model of ref. [20] which was derived from a WZW–model.
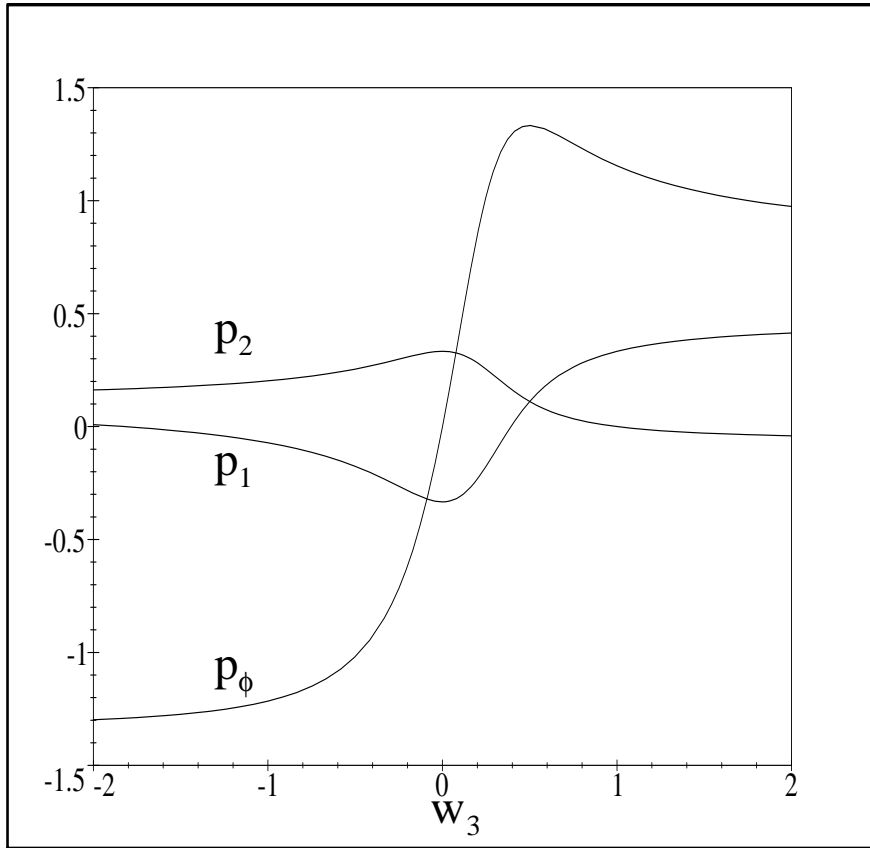
Fig 1: Expansion coefficients for the (+) branch at $t \simeq t_0$.

The asymptotic regions where the form is effectively turned off are specified by $\tau \simeq 0$ and $\tau \simeq \tau_1$ and the corresponding expansion coefficients in

eq. (1) read

$$
\tau \simeq 0 \quad : \quad p_I^{(0)} = \frac{w_I}{x}
$$

$$
\tau \simeq \tau_1 \quad : \quad p_I^{(1)} = \frac{w_I - c_I}{x + \Delta} \ . \tag{14}
$$

After using the constraints (8) they still depend on one free parameter, which here we take to be w_3. An example of this dependence for the $(+)$ branch is given in fig. 1 and fig. 2. It can be seen that the 3–dimensional subspace which for small w_3 is contracting at early times $t \simeq t_0$ is turned to expansion at late time $t \to \infty$.

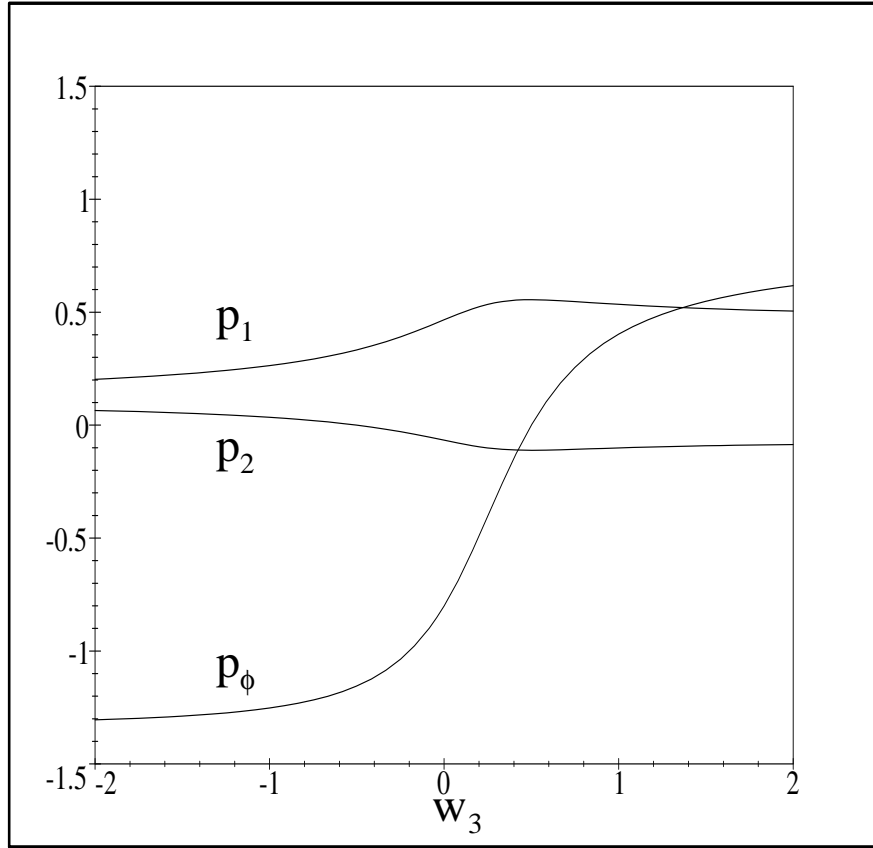*Fig 2: Expansion coefficients for the $(+)$ branch at $t \to \infty$.*

The converse is true for the 6–dimensional "internal" space. In the $(-)$ branch an early $t \to -\infty$ contraction of the 3–dimensional subspace and a hyperinflating expansion of the 6–dimensional subspace can similarly be turned into its converse as t approaches t_1. Though this "desirable" situation is clearly arranged in the present context by choosing initial conditions in the correct range it is by no means obvious that this was possible at all. In a very similar model with a $(d_1, d_2) = (3, 6)$ or $(d_1, d_2) = (3, 7)$ split but an *elementary* 3–form in the $d_1 = 3$ subspace, to be realized in the context of type IIA or 11–dimensional supergravity, it is impossible to have 3 expanding and 6 or 7 contracting dimensions as $t \to \infty$.

Next, we would like to study how the above picture generalizes once two forms are turned on. We choose a split $(d_1, d_2, d_3) = (3, 2, 4)$, with an elementary IIA 3–form $F^{(3)}$ in the $d_1 = 3$ subspace and a solitonic IIA 1–form $F^{(1)}$ in the $d_2 = 2$ subspace. This results in the following Ansatz :

$$
\begin{aligned}
ds^2 &= -N^2(\tau)d\tau^2 + e^{2\alpha_1} \sum_{\mu=1}^{3} dx_\mu^2 + e^{2\alpha_2} \sum_{m=4}^{5} dx_\mu^2 + e^{2\alpha_3} \sum_{a=6}^{9} dx_\mu^2 \\
F^{(3)}_{0\mu\nu\rho} &= e^{-6\alpha_1} h_1' \, \epsilon_{\mu\nu\rho} \\
F^{(1)}_{mn} &= e^{-4\alpha_2} v_2 \, \epsilon_{mn} \\
\phi &= \phi(\tau) \, .
\end{aligned}
\tag{15}
$$

The model simplifies considerably in the harmonic gauge $N = \exp(3\alpha_1 + 2\alpha_2 + 4\alpha_3)$ where it corresponds to a Hamiltonian system. The equation of motion for $F^{(3)}$ can be integrated to give $h_1' = v_1 \exp(6\alpha_1 - \phi/2)$ with a constant v_1. This result can be used to replace h_1' in the other equations. To simplify the notation it is useful to introduce the vector $\boldsymbol{\alpha} = (\alpha_I) = (\alpha_i, \phi)$ and the metric G_{IJ} with $G_{ij} = 2(d_i \delta_{ij} - d_i d_j)$, $G_{i\phi} = G_{\phi i} = 0$, $G_{\phi\phi} = 8/(D-2)$. The equations of motion for $\boldsymbol{\alpha}$ can be obtained by a variation of the Lagrangian

$$
\begin{aligned}
\mathcal{L} &= \frac{1}{2} \boldsymbol{\alpha}'^T G \boldsymbol{\alpha}' - U \\
U &= \frac{v_1^2}{2} \exp(\mathbf{q}_1.\boldsymbol{\alpha}) + \frac{v_2^2}{2} \exp(\mathbf{q}_2.\boldsymbol{\alpha})
\end{aligned}
\tag{16}
$$

The influence of the forms on the effective potential U is encoded in the vectors $\mathbf{q}_1 = (6, 0, 0, -1/2)$ and $\mathbf{q}_2 = (6, 0, 8, 3/2)$ for the 3– and the 1–form

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respectively. The above system has to be supplemented by the Hamiltonian constraint

$$\mathcal{H} = \frac{1}{2}\boldsymbol{\alpha}'^T G \boldsymbol{\alpha}' + U = 0 \ . \tag{17}$$

Contact with Toda theory can be made if the matrix $< \mathbf{q}_r, \mathbf{q}_s >$ computed with the scalar product defined by $< \mathbf{x}, \mathbf{y} >= \mathbf{x}^T G^{-1}\mathbf{y}$ is proportional to the Cartan matrix of a semi–simple Lie group. In fact, in our case we have $< \mathbf{q}_1, \mathbf{q}_1 >=< \mathbf{q}_2, \mathbf{q}_2 >= 4$ and $< \mathbf{q}_1, \mathbf{q}_2 >= 0$ which corresponds to the Cartan matrix of $SU(2)^2$. This allows one to decouple the two exponentials in the potential U and to find the explicit solution [16]

$$\boldsymbol{\alpha} = \sum_{r=0}^{3} \rho_r G^{-1}\mathbf{q}_r \tag{18}$$

with $\mathbf{q}_1$, $\mathbf{q}_2$ as above, $\mathbf{q}_0 = (3, 1, -8, 0)$, $\mathbf{q}_3 = (4, 0, 8, 1)$ and the functions

$$\rho_r = -\frac{1}{4}\ln\left(\frac{2v_r^2}{k_r^2}\cosh^2(|k_r|(\tau - \tau_r))\right) \ , \quad r = 1, 2 \tag{19}$$

$$\rho_r = k_r(\tau - \tau_r) \ , \quad r = 0, 3 \ . \tag{20}$$

The time parameter τ ranges over the full real axis and k_r, τ_r, $r = 0...3$ are integration constants. The Hamiltonian constraint (17) turns into

$$k_1^2 + k_2^2 + \frac{8}{3}k_3^2 = \frac{1}{4}k_0^2 \ . \tag{21}$$

As in the first example this condition can be used to prove the existence of a $(+)$ and a $(-)$ branch with general properties as discussed in the introduction. The two Eigenmodes ρ_1 and ρ_2 describe the effect of the 3– and the 1–form, respectively. The corresponding integration constants τ_1 and τ_2 have a very simple interpretation as can be seen from eq. (19). The 3–form is operative around $\tau \simeq \tau_1$, the 1–form around $\tau \simeq \tau_2$. This allows for three asymptotic Kaluza–Klein regions $\tau \to -\infty$, $\tau_1 \ll \tau \ll \tau_2$ (or $\tau_2 \ll \tau \ll \tau_1$) and $\tau \to +\infty$ where the forms are effectively turned off. The two forms act independently, each being responsible for one of the two transitions. In this sense, the model can be interpreted as a time sequence of two simple models each with just one form turned on.

Finally, we would like to discuss an example which does not lead to a "decoupling" of the two forms in the above sense. We start with the same

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Ansatz (15) as in the previous example except for the solitonic RR 1–form $F^{(1)}$ which we replace by an elementary NS 2–form $F^{(2)}$ in the same $d_2 = 2$ subspace and with the Ansatz

$$F^{(2)}_{0mn} = e^{-4\alpha_2} h'_2 \, \epsilon_{mn} \; . \tag{22}$$

h The equation of motion for $F^{(2)}$ is used to replace h'_2 by $h'_2 = v_2 \exp(4\alpha_2 + \phi)$ and in the harmonic gauge we end up with the same system specified by (16) and (17) but with $\mathbf{q}_2$ replaced by $\mathbf{q}_2 = (0, 4, 0, 1)$. Now the scalar products turn out to be $< \mathbf{q}_1, \mathbf{q}_1 >=< \mathbf{q}_2, \mathbf{q}_2 >= 4$ and $< \mathbf{q}_1, \mathbf{q}_2 >= -2$ indicating that we are dealing with an $SU(3)$ Toda theory. The solution [13, 16] takes the form (18) with ρ_0 and ρ_3 given by eq. (20) and $\mathbf{q}_0 = (1, 2/3, 1, 0)$, $\mathbf{q}_3 = (0, -4/3, 0, 1)$. For ρ_1 and ρ_2 we get

$$\rho_r = -\frac{1}{2} \ln \left(\sum_{\boldsymbol{\lambda} \in \Lambda_r} b_r(\boldsymbol{\lambda}) \exp(\boldsymbol{\lambda}.\mathbf{k}\,\tau - \boldsymbol{\lambda}.\boldsymbol{\tau}) \right) , \quad r = 1, 2 , \tag{23}$$

where Λ_r are the weight systems of the two fundamental $SU(3)$ representations, namely $\Lambda_1 = \{(1, 0), (-1, 1), (0, -1)\}$ for $\mathbf{3}$ and $\Lambda_1 = \{(0, 1), (1, -1), (-1, 0)\}$ for $\bar{\mathbf{3}}$ and $\mathbf{k} = (k_1, k_2)$, $\boldsymbol{\tau} = (\tau_1, \tau_2)$. The coefficients $b_r(\boldsymbol{\lambda})$ can be expressed as

$$
\begin{array}{llcllll}
b_1(1, 0) & = & 2v_1^2 \frac{2k_2 - k_1}{D} & \quad & b_2(0, 1) & = & 2v_1^2 \frac{2k_1 - k_2}{D} \\
b_1(-1, 1) & = & 2v_1^2 \frac{k_1 - k_2}{D} & \quad & b_2(1, -1) & = & 2v_2^2 \frac{k_1 - k_2}{D} \\
b_1(0, -1) & = & 2v_2^2 \frac{2k_1 - k_2}{D} & \quad & b_2(-1, 0) & = & 2v_2^2 \frac{2k_2 - k_1}{D}
\end{array}
\tag{24}
$$

with $D = (k_1 - 2k_2)(k_1 - k_2)(2k_1 - k_2)$. The Hamiltonian constraint now reads

$$k_1^2 - k_1 k_2 + k_2^2 + \frac{4}{3} k_3^2 = \frac{1}{24} k_0^2 \tag{25}$$

and we demand $2k_1 - k_2 > 0$, $2k_2 - k_1 > 0$ ($\mathbf{k}$ is in the Weyl chamber) to ensure a positive argument of the logarithm in eq. (23). The picture of the previous example applies in this case as well. For $\tau \to \pm\infty$ we have two asymptotic Kaluza–Klein regions and depending on the choice of integration constants two intermediate Kaluza–Klein regions can occur.

In conclusion, we have presented a first study of cosmological solutions with nontrivial RR–fields in type II string theory. We have shown that models with one RR–form are generally solvable and a number of interesting

examples with two forms can be solved because they correspond to Toda models. It turns out that the potential provided by the RR–forms generates interesting dynamics which allows interpolation between different pure Kaluza–Klein states. A key ingredient to a direct cosmological application of our results is certainly to connect the hyperinflating and the subluminal branch which we found in all our solutions. This turned out to be impossible within the class of models we have analyzed. It is likely that there is no classical way at all to achieve such a connection and that nonperturbative physics has to be invoked for that. As for pre–big–bang models which suffer from the same problem one might speculate that a duality map between the branches corresponds to the correct transition mechanism.

Our Ansatz and the solution (7) turned out to be very close to membrane solutions which might suggest the possibility of a cosmological BPS state preserving half of the supersymmetries. Such a solution would certainly have very interesting applications to density fluctuations in the early universe which presumably could be "counted" in a similar way as the states of some extremal black holes [21].

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